Fraction Competency Packet

Student Support
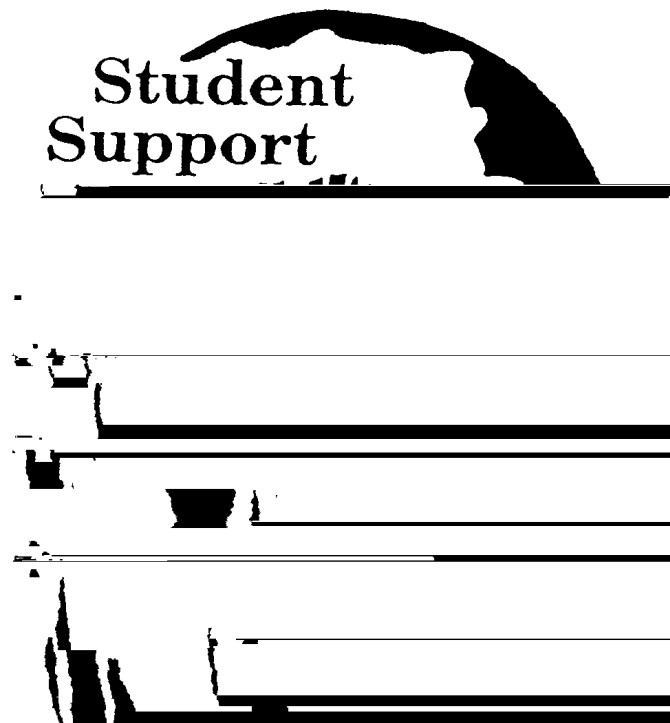
Developed by: Nancy Tufo

To use this booklet, review the glossary, study the examples, then work through the exercises. The answers are at the end of the booklet. When you find an unfamiliar word, check the glossary for a definition or explanation.

Calculators are not allowed when taking the Computerized Placement Test (CPT), nor in Fundamentals of Mathematics, Pre-Algebra, and Elementary Algebra; therefore, do not rely on a calculator when working the problems in this booklet.

If you have difficulty understanding any of the concepts, come to one of the Tutoring Centers located on the Lynn, Danvers Main and Danvers Hathorne Campuses. Hours are available at (978) 762-4000 x 5410. Additional Tutoring Center information can be found on the NSCC website at www.northshore.edu/services/tutoring. The Centers are closed when school is not in session, and Summer hours are limited.

Table of Contents

Glossary

Boosting: Rewriting a fraction as an equivalent fraction with a higher denominator.

Denominator:

Proper Fraction: Any fraction when the numerator is less than the denominator.

Quotient: The solution to a division problem.

Reducing: Dividing the numerator and the denominator by the same number to get an equivalent fraction. Final answers of most fraction problems should be expressed reduced to "simplest terms"; in other words, the numerator and denominator have no more common factors.

Remainder: The number left after a whole number division problem is complete. When converting an improper fraction to a mixed number, the remainder is the numerator of the fraction.

Sum: the result when two numbers are added.

Whole Number: The Numbers system including 0, 1, 2, 3,….

General Fraction Information
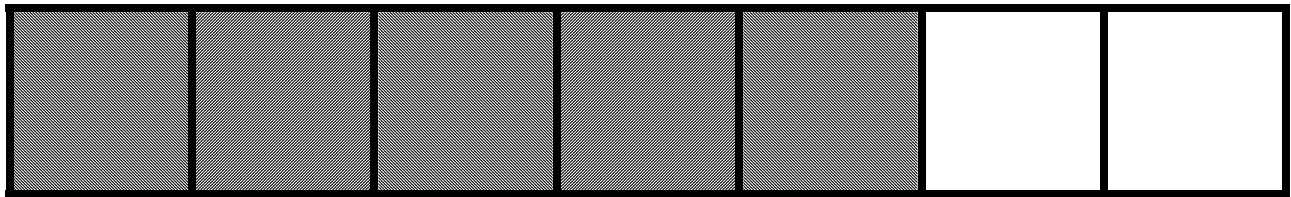
Ø   The fraction that represents the above picture is $\dfrac{5}{7}$ and is read "five sevenths". That means that five of the parts are shaded, and it would take seven parts of that size to make a whole.

Ø   One whole can be "cut up" into equal size parts; therefore, $1 = \dfrac{13}{13} = \dfrac{9}{9} = \dfrac{123}{123}$, etc.

Ø   A whole number can be written as a fraction with a denominator of 1; for example, $2 = \dfrac{2}{1}$. Zero can be written as a fraction using zero as the numerator and any whole number as the denominator, for example, $\dfrac{0}{23}$.

Ø   Any whole number may be written as a mixed number by using a zero fraction. For example, $3 = 3\dfrac{0}{42}$.

Mixed Numbers

To convert a mixed number, $5\frac{2}{7}$, to an improper fraction, $\frac{37}{7}$:

$$5\frac{2}{7} = \frac{37}{7}$$

$5\frac{2}{7}$	Work in a clockwise direction, beginning with the denominator, (7).
5 x 7 = 35	Multiply the denominator (7) by the whole number, (5)
35 + 2 = 37	Add that product, (35), to the numerator (2) of the fraction.
$\frac{(5\times7)+2}{7} = \frac{37}{7}$	The denominator remains the same for the mixed number and the improper fraction.

Convert to Improper Fractions:

1) $4\frac{2}{5} =$

6) $14\frac{3}{4} =$

11) $9 =$
    Hint: See #10

2) $\frac{3}{8} =$

Finding Equivalent Fractions with Larger Denominators
This process is sometimes called "**Boosting**"

$$Example: \frac{5}{8} = \frac{?}{56}$$

$56 \div 8 = 7$ — Divide the larger denominator by the smaller to find the factor used to multiply the denominator. (Note: The product of the smaller denominator and the factor is the larger denominator)

$\frac{5}{8} \times \frac{7}{7} = \frac{5 \times 7}{8 \times 7}$ — Use this factor to multiply the numerator.

$\frac{5}{8} = \frac{35}{56}$ — The result is two equivalent fractions.

Note: Equal denominators are required for addition and subtraction of fractions.

Find the equivalent fractions as indicated:

1) $\frac{2}{5} = \frac{}{15}$

2) $\frac{3}{8} = \frac{}{32}$

3) $\frac{4}{9} = \frac{}{54}$

4) $\frac{6}{7} = \frac{}{49}$

5) $\frac{1}{8} = \frac{}{48}$

6) $\frac{3}{4} = \frac{}{44}$

7) $\frac{3}{5} = \frac{}{45}$

8) $\frac{1}{10} = \frac{}{60}$

9) $\frac{1}{2} = \frac{}{28}$

10) $\frac{10}{100} = \frac{}{700}$

11) $\frac{8}{9} = \frac{}{81}$

12) $\frac{3}{4} = \frac{}{68}$

13) $\frac{5}{9} = \frac{}{108}$

14) $\frac{3}{8} = \frac{}{112}$

15) $\frac{2}{3} = \frac{}{462}$

Equivalent Fractions with Smaller Denominators
Reducing Fractions

Example: Reduce the following fraction to lowest terms

$$\frac{90}{105}$$

There are **three** common **methods,** DO NOT mix steps of the methods!

Method 1:

$$\frac{90 \div 15}{105 \div 15} = \frac{6}{7}$$

The Greatest Common Factor for 90 and 105 is 15. Divide the numerator and the denominator by the GCF, 15.

Method 2:

$$\frac{90 \div 5}{105 \div 5} = \frac{18}{21}$$

$$\frac{18 \div 3}{21 \div 3} = \frac{6}{7}$$

Examine the numerator and denominator for any common factors, divide both numerator and denominator by that common factor. Repeat as needed.

Ø Both 90 and 105 are divisible by 5.

Ø Both 18 and 21 are divisible by 3.

Method 3:

$$\frac{90}{105} = \frac{2 \times 3 \times 3 \times 5}{7 \times 3 \times 5}$$

Express the numerator and denominator as a product of prime factors.

$$\frac{90}{105} = \frac{2 \times 3 \times (3 \times 5)}{7 \times (3 \times 5)}$$

Divide numerator and denominator by common factors, (3x5)

$$= \frac{2 \times 3}{7} = \frac{6}{7}$$

Multiply remaining factors.

Reduce these fractions.

1) $\dfrac{28}{50} =$

2) $\dfrac{8}{24} =$

3) $\dfrac{30}{54} =$

4) $\dfrac{18}{42} =$

5) $\dfrac{32}{48} =$

6) $\dfrac{36}{54} =$

7) $\dfrac{14}{56} =$

8) $\dfrac{18}{28} =$

9) $\dfrac{36}{216} =$

10) $\dfrac{35}{42} =$

11) $12\dfrac{54}{99} =$

12) $15\dfrac{280}{320} =$

8

Improper Fractions

Example: Convert $\dfrac{14}{3}$ to an Improper Fraction

$14 \div 3 = 4$ Remember: Dividend $\div$ Divisor = Quotient
Remainder 2   Divide the numerator (14) by the denominator (3).

$\dfrac{14}{3} = 4\dfrac{2}{3}$ Write the mixed number in the form: $Quotient \dfrac{remainder}{divisor}$

Note: Check you answer to see if you can reduce the fraction.

Convert these improper fractions to mixed numbers. *Be sure to reduce when it's possible.*

1) $\dfrac{8}{5} =$

6) $\dfrac{114}{5} =$

2) $\dfrac{18}{7} =$

7) $\dfrac{128}{3} =$

3) $\dfrac{37}{9} =$

8) $\dfrac{401}{3} =$

4) $\dfrac{127}{5} =$

9)   36c(8))Tj2.5751 0.6738 SBh -1.16746b9Bh 13.98 141.5 (

5) $\dfrac{32}{9} =$

9

Least Common Multiple (LCM)
Used to find the Least Common Denominator (LCD)

Example: Find the LCM of 30 and 45

Note: There are **four** common **methods**; DO NOT mix the steps of the methods!

Method 1

Remember that multiples are equal to or larger than the given number.

30, 60, **90**, 120, …   List the multiples of each of the given numbers, in
45, **90**, 135, …   ascending order.

       $LCM = 90$ The LCM is the first multiple common to both lists.

Method 2

45, 90, 135, …   List the multiples of the larger number.

$45 \div 30$ remainder   Divide each in turn by the smaller.

$90 \div 30$ no remainder   The LCM is the multiple that the smaller number
       $LCM = 90$ divides without leaving a remainder.

Method 3

$30 \div 5 = 6$; $45 \div 5 = 9$
$6 \div 3 = 2$; ~~9~~   3   3 $\div$

In each exercise, find the LCM of the given numbers.

1)   4   and   18   7)   50   and   75

2)   16   and   40   8)   24   and   30

3)   20   and   28   9)   36   and   45

4)   5   and   8   10)   8   and   20

5)   12   and   18   11)   16   and   20

                                                      12)   28,   35,   and   21

6)   12   and   16

Addition and Subtraction of Fractions
with the Same Denominator

To add or subtract fractions, the denominators MUST be the same.

Example 1:

$$\frac{3}{5} - \frac{1}{5} = ?$$

$$\frac{3}{5} - \frac{1}{5} = \frac{3-1}{5}$$

Because both fractions have the same denominator, you may subtract the numerators and keep the denominator.

$$= \frac{2}{5}$$

Example 2:

$$\frac{5}{9} + \frac{7}{9} = ?$$

$$\frac{5}{9} + \frac{7}{9} = \frac{5+7}{9}$$

Because both fractions have the same denominator, you may add the numerators and keep the denominator.

$$= \frac{12}{9}$$

$$= 1\frac{3}{9}$$

Always change improper fractions to a mixed number.

$$= 1\frac{1}{3}$$

Reduce, when possible.

Add or Subtract as indicated.

1. $\dfrac{4}{8} + \dfrac{3}{8}$

2. $\dfrac{7}{10} - \dfrac{1}{10}$

3. $\dfrac{7}{48} + \dfrac{9}{48} + \dfrac{4}{48}$

4. $\dfrac{40}{37} - \dfrac{3}{37}$

5. $\dfrac{10}{13} + \dfrac{4}{13}$

6. $\dfrac{9}{17} + \dfrac{11}{17} + \dfrac{17}{17}$

7. $\dfrac{2}{3} + \dfrac{4}{3} - \dfrac{6}{3}$

8. $\dfrac{7}{6} - \dfrac{5}{6} + \dfrac{1}{6}$

9. $\dfrac{7}{13} + \dfrac{9}{13}$

Addition and Subtraction of Fractions
with Different Denominators

Remember: In order to add or subtract fractions, the denominators MUST be the same.

Example:

$$\frac{2}{3} + \frac{3}{8} = ?$$

LCM = 24   Find the LCM

$$\frac{2}{3} \times \frac{8}{8} = \frac{16}{24}$$

$$+ \quad - \quad - \quad \frac{9ss24}{24}$$

Subtraction of Fractions with Borrowing

Subtract:

Multiplication of Fractions

Example:

$$\frac{3}{10} \times 3\frac{5}{6}$$

Note: LCD is not needed to multiply fractions.

$3\frac{5}{6} = \frac{(6\times3)+5}{6}$ Change mixed numbers to improper fractions

$\frac{3}{10} \times \frac{23}{6} = \frac{1\times23}{10\times2}$ Before multiplying, reduce by dividing any numerator with any denominator with a common factor. (3 and 6 have a common factor of 3)

$\frac{1\times23}{10\times2} = \frac{23}{20}$ Multiply numerators and denominators

$\frac{23}{20} = 1\frac{3}{20}$ Convert improper fractions to mixed numbers.

Multiply:

1) $4\frac{1}{2} \times \frac{2}{3}$ 5) $\frac{10}{11} \times 1\frac{7}{15}$ 9) $9\frac{7}{8} \times \frac{4}{5}$

2) $3\frac{1}{5} \times 1\frac{1}{4}$ 6) $4\frac{3}{5} \times 15$ 10) $7\frac{9}{10} \times 1\frac{1}{4}$

3) $6 \times 1\frac{1}{9}$ 7) $3\frac{3}{8} \times 2\frac{2}{9}$ 11) $18 \times 1\frac{3}{7} \times \frac{4}{15}$

4) $2\frac{1}{6} \times 1\frac{1}{2}$ 8) $34 \times 2\frac{3}{17}$ 12) $3\frac{1}{5} \times 1\frac{5}{6} \times \frac{3}{8}$

Division of Fractions

Example:

$$2\frac{3}{4} \div 2\frac{3}{8} \qquad \textbf{OR} \qquad \frac{2\frac{3}{4}}{2\frac{3}{8}}$$

Note: One fraction divided by another may be expressed in either way shown above. Also, LCD is not needed to divide fractions.

$2\frac{3}{4} = \frac{11}{4}$ and $2\frac{3}{8} = \frac{19}{8}$ Convert mixed numbers to improper fractions

$\frac{11}{4} \div \frac{19}{8} = \frac{11}{4} \times \frac{8}{19}$ Invert the divisor $\frac{19}{8}$. (Turn the fraction after the division sign upside down)

$\frac{11 \times 8}{4 \times 19} = \frac{11 \times 2}{1 \times 19}$ Reduce if possible. (4 and 8 have a common factor)

$\frac{11 \times 2}{1 \times 19} = \frac{22}{19}$ Multiply numerators and denominators

$\frac{22}{19} = 1\frac{3}{19}$ Convert to a mixed number and reduce if needed.

Divide these fractions.   Reduce to lowest terms!

1) $\frac{5}{6} \div \frac{1}{2}$

2) $\frac{3}{4} \div \frac{3}{7} =$

3) $3 \div 1\frac{2}{5} =$

4) $\dfrac{\frac{1}{2}}{\frac{1}{3}} =$

5) $\frac{1}{2} \div 6 =$

6) $2\frac{1}{4} \div 3 =$

7) $3\frac{1}{7} \div 2\frac{5}{14} =$

8) $\dfrac{2\frac{5}{8}}{1\frac{7}{8}}$

9) $4\frac{1}{2} \div 1\frac{3}{4} =$

17

Solve the following problems.

1. An empty box weighs $\dfrac{1}{4}$

p. 12	p. 13	p. 15	p. 16	p. 17
1) $\frac{7}{8}$	1) $1\frac{5}{8}$	1) $2\frac{2}{3}$	1) 3	1) $1\frac{2}{3}$
2) $\frac{3}{5}$	2) $\frac{1}{8}$	2) $5\frac{5}{6}$	2) 4	2)